Quantum Networks: Reset-and-Reuse can be a Game-changer for Entanglement via Distillation

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Abstract— The future quantum network repeater is envisioned to primarily serve the role of creating entanglement between nodes and distilling those entanglements to an optimal level of performance. During our investigation, we implemented a multi-pass protocol for entanglement distillation and tested it on the IBM-Q environment, demonstrating successively improved results after multiple passes. We implemented two versions of multi-pass distillation, BBPSSW and DEJMPS, with a focus on optimizing the use of qubits, via the reset-and-reuse capability of the IBM implementation. The novel feature of reset-and-reuse can be a game-changer and can minimize the number of qubits required for large-scale applications. We also found that, though it is currently not possible to implement a criterion for continued distillation passes as a run-time feedback loop, the process can be studied through post-circuit data analysis. Our results also show that fidelity alone may guide us to discard some approaches that show success based on other metrics, such as entanglement success and success of transmitting a bit of data. The fidelity was experimentally found to be excessively low, for this complex process of multi-pass distillation. (*Abstract*)

Keywords—quantum, Networking, Entanglement, Distillation, Multi-pass, Qiskit (keywords)

I. POSTER RELEVANCE

To our knowledge, no experimental implementations or analyses have been done, of distillation as a multi-pass process.

For our multi-pass distillation implementation, we utilized the reset and reuse functionality in Qiskit SDK, which allows a qubit to be reallocated mid-circuit, freeing up the resource for other uses. Reset and reuse worked exceptionally well for multi-pass distillation, since two of the four qubits are only used temporarily as ancillary qubits. Using reset, our implementation of multi-pass entanglement distillation requires only five qubits (four for distillation plus one data qubit), regardless of the number of passes. Without the reset functionality, implementing a multi-pass protocol would require an additional entangled pair of qubits for each pass.

In this paper we implemented a post-circuit data analysis approach, to enable experimental testing of multi-pass quantum distillation, given the current limitation of IBM quantum systems (the only variant tested in this research). Post-circuit execution data enabled us to characterize the distillation process in terms of optimizing the number of passes and parameters used.

Fidelity, a measure of how closely the final experimental quantum state (calculated using tomography) matches the theoretical quantum end state of these complex distillation circuits was found to have a poor correlation. In contrast, these same circuits had high entanglement success and high success passing a data bit via teleportation. These results point to some limitations of fidelity as a sole means to characterize the effectiveness of a quantum circuit.

II. EXTENDED POSTER ABSTRACT

A. Introduction

Quantum technology advancement will require node-to-node network communications. Whether those nodes are local to a machine or separated between sites, across a long distance, the bond that binds them will be quantum entanglement. Entanglement is a link between quantum bits (qubits) where the data value (state) is constantly in sync, regardless of how far apart they are. A change of state from one qubit will be immediately reflected in the other qubit(s) entangled with it.
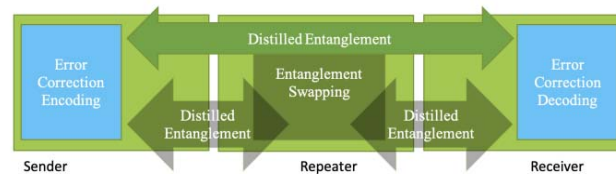

Fig. 1. The quantum networking process: 1) the repeater will establish and distill entanglement between each sender and receiver (or an adjacent repeater) independently. 2) Once entanglement is established, entanglement swapping will replace the point-to-point connections with the repeater to a direct entangled link between the sender and receiver. Distillation is the primary topic of this research, with a focus on optimization of each of these links, given the near-term limitations of qubits quantity and reliability.

For teleportation, the transfer of quantum information via entanglement, to be successful we must optimize the strength and resilience of the underlying entanglement. Refining entanglement, through the process of entanglement distillation [2], will be a key to success. For quantum networks, repeaters will be required to establish the long-distance, end-to-end connection to teleport quantum data via entanglement. These quantum repeaters will differ, in their role, compared to digital

telecommunications routers and switches we utilize today. The role of the repeater is to create robust entanglement "pathways". Per the Internet Engineering Task Force's (IETF) Quantum Internet Working Group [1], quantum repeaters as currently envisioned, will not access, transmit, amplify, or correct the user data. The repeater will instead be focused on creating entanglement between nodes and distilling (refining) those entanglements to an optimal level. The diagram in figure 1 represents the process of entanglement for the quantum network.

Once entanglement is established, it can be further refined through the process of single or multi-pass entanglement distillation, a process that transforms some number of entangled pairs into a smaller number of more maximally entangled pairs. As previously envisioned, full multi-pass distillation protocols were impractical due to limitations of the current generation of quantum hardware, because the process: 1) is expected to include a feedback loop and proceed only if the previous attempt of distilled entanglement was successful (reading feedback during a circuit is not currently possible on the IBM quantum implementation), 2) would require a feedback loop to determine if further distillation passes are required, to reach the target fidelity (determining fidelity, via tomography, and reading back the result is not possible in current technology), 3) would require additional qubits for each pass, which is impractical, given the limited resources available for experimentation.

As quantum computers mature, we need to overcome the current limitations continually and creatively, until more advanced features are implemented in the hardware. Prior to 2021, the IBM quantum system implementation limited output from a program's circuit to just one bit of data per qubit. Given the small number of qubits then and still today, this limitation had an enormous negative impact on realizing many algorithms and processes. In early 2021 IBM announced a new feature, mid-circuit reset. This feature enables output from any qubit to a classical bit at any step in the circuit. The measured qubit can then be reset, to a value of 0, for reuse within the circuit. This change effectively provided the means to write an unlimited number of output bits during a circuit, even when the circuit is run on a small machine with just a few qubits. This feature effectively expands the number of qubits available for operations due to the reusability of qubits during a circuit

Now that the output limitation has been overcome, there still does not yet exist an ability to use (read) those values that have been saved to classical bits as input to the circuit. The simulator is capable of this, indicating the possibility of such a future hardware feature, but the current machines are not. This limitation at first glance would seem to limit the implementation and testing of multi-pass processes, on real quantum hardware, if the decision to run another pass is dependent on the outcome saved to the classical bits from the previous pass(es).

B. Entanglement Distillation

Optimally entangled links will be required, to better withstand long-distance quantum communications environments. Entanglement distillation is a refinement process, designed to "strengthen" the entanglements by combining less entangled pairs of qubits into a single more optimally entangled pair. Long-distance quantum networks will need highly resilient entanglement links to reliably pass data.

Experiments of single-pass distillation have been well documented [2,3,4]. Less studied, though mentioned in literature [5], was the expectation that distillation would be repeated, until an optimized level of entanglement is achieved, strengthening the entanglement during each pass. Our efforts were focused on designing and testing multi-pass distillation circuits to quantitatively evaluate the impact of the process.

At the end of each distillation pass, auxiliary bit values need to be compared, to determine if entanglement was successful and whether the process should continue to the next pass. If the entanglement is unsuccessful, the process would be stopped and restarted from the beginning.

Techniques for distillation had been developed, before the implementation of actual quantum systems including BBPSSW distillation, developed in 1996 by Bennett-Brassard-Popescu-Schumacher-Smolin-Wootters [6], and DEJMPS distillation, developed in 1996 by Deutsch-Ekert-Jozsa-Macchiavello-Popescu-Sanpera [4].

The BBPSSW implementation we used for testing was based on IBM Qiskit information and an award-winning Qiskit tutorial [2,3], with modifications (i.e., to enable multi-pass distillation and optimized for real quantum hardware, etc.). The BBPSSW protocol places the four qubits into a depolarized Werner state [3], prior to distillation, by manipulating the entangled qubits with one of twelve, randomly selected, sets of bilateral rotation gates.
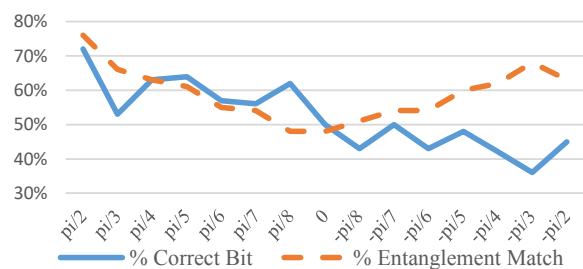

Fig. 2. BBPSSW Distillation: Analysis of parameter theta used to calculate incoming fidelity, shows teleported data correctness was optimized as $\pi/2$ while successful entanglement was most successful at values approaching $\pi/2$ and -$\pi/2$.

In the BBPSSW tutorial [3], varying a parameter of theta adjusted as a simulated fidelity value (in-circuit feedback of fidelity is not currently possible on a quantum system, the inclusion in the tutorial was used to demonstrate the capability of using a parameterized value). During our testing on quantum hardware, we noted that a fixed value of theta of $\pi/2$ generated optimal results (Fig. 2) as far as both entanglement success and

teleporting a bit value, hence we included $\pi/2$ in our circuit design, as an additional means of optimization.

C. Multi-pass Distillation using Qubit Reset and Reuse

Quantum distillation literature [2,4] discusses the expectation that the distillation process would be repeated, resulting in more maximally entangled qubits after each pass of a multi-pass process.

For the two ancillary qubits that are measured and stored to classical bits at the completion of each pass in the circuit, matching values of zero or one is indicative that the remaining two qubits are entangled. During data analysis, we can identify the specific classical bits related to each pass and discard the data for that attempts that fail to entangle during any of the multiple passes. We also analyzed the results at each pass, to determine what the success of entanglement is after a given number of passes.

Fig. 3 shows an example of a 3-pass implementation of the BBPSSW circuit using reset to reuse for the ancillary qubits after each pass. We see vertical grey barriers that delineate various steps in the circuit. First, we see the four qubits used for distillation being initialized to the value zero and "conditioned" with the U and X gates. "Twirling" occurs following the first barrier and the ancillary qubits are measured and reset to the value of zero. After the 2nd barrier, during the 2nd pass, the ancillary bits are reconditioned, "twirled", entangled with the two primary qubits from the first pass, and measured and reset. This process is repeated for each pass. After the sixth barrier, we see the data bit (on qd0) being initialized to a value of one and (after the seventh barrier) being transferred to the entangled pair. We then measure one of the primary entangled qubits (qe1) to see if the value of 1 has been successfully teleported over the entangled pair.
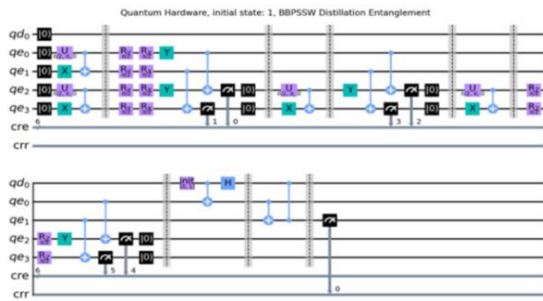

Fig. 3. BBPSSW 3-pass Distillation circuit with reset, generated via IBM's Qiskit SDK implemented via Python code. This circuit transmits an initialized value of one from qubit qd0 to qe1 (representing a receiver's qubit).

D. Single and Multi-pass Distillation Results

Circuits were verified by running tests on the quantum simulator and achieving 100% expected results for both entanglement success and passing a data bit.

During the experiments run on the quantum computers, we found that approximately 20% of circuit runs were unsuccessfully distilled during each pass, of the multi-pass distillation circuit. In Table 1, we show the percent of runs that

were successfully entangled, (where measured values were both zero or both one) after each pass. No attempt to pass the data bit was done during this round of testing. We note that the number of practical distillation passes may be self-limiting, as we need an adequate quantity of successful runs to produce the data needed to statistically evaluate the results of successful data bit transmission.

Table 1. Entanglement Success Rate Based on Distillation Passes

Protocol	Pass	1	2	3	4	5
BBPSSW	1	58%				
BBPSSW	2	87%	66%			
BBPSSW	3	38%	25%	20%		
BBPSSW	4	83%	63%	54%	40%	
BBPSSW	5	78%	53%	32%	27%	21%
DEJMPS	1	86%				
DEJMPS	2	82%	68%			
DEJMPS	3	87%	71%	60%		

After discarding the data associated with the experimental runs that were unsuccessful at achieving entanglement during any pass (as indicated when measured ancillary qubit values do not match), we analyzed the success of transmitting a data bit via the successfully distilled entanglements. Figure 4 shows these test results for BBPSSW and DEJMPS distillation protocol, with one to eight passes. The output was analyzed for data transmission success. As a baseline, we are comparing results to a single two-qubit Bell entangled pair, which was the most successful process, though expected to be far too fragile for quantum network use in production. BBPSSW appears to improve up to through the 4th pass on this set of data. DEJMPS appeared to fall after the 2nd pass and no additional passes were tested.
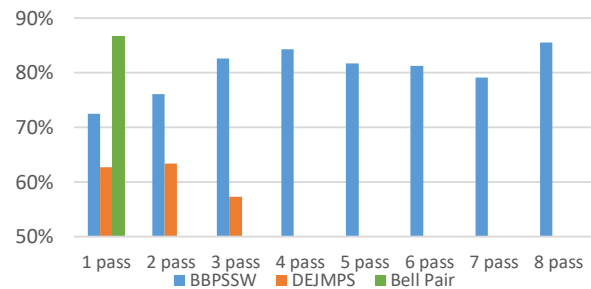

Fig. 4. Multi-pass Distillation data bit transmission success (passing a value of one). We can see the BBPSSW protocol rises steadily during the first four passes.

Another common measure used to compare quantum circuits is fidelity. For circuits up to five qubits, it is practical to determine fidelity using the process of tomography, which compares the theoretical final density state of the circuit to an experimentally reconstructed final density state. During our testing, we tried to limit the number of qubits used to five, to support the use of tomography.

The tomography process attempts to overcome the inability to measure the complex final state experimentally (as any

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attempt to measure the qubits collapsed each to a value of either a one or zero), by rerunning our distillation circuit thousands of times, via a tomography function available in Qiskit, and measuring all the qubits from many different global phases. From these results, tomography attempts to reconstruct the final state of the qubit set. With the theoretical and experimental tomography results calculated, fidelity can be determined, as a result between zero (meaning no correlation between the experimental and the theoretical density matrix) and one (perfect correlation).

To more easily visualize the fidelity phenomenon comparison (theoretical to experimental), we present the data with the real and imaginary parts of the state matrix plotted like a city (city-state diagrams). The following diagrams show the 3-pass BBPSSW circuit. In figure 5, the theoretical density matrix displays well-formed probability peaks of the expected results. Our experimental city-state diagram results (Fig. 6) show vaguely higher probabilities along the diagonals, as expected, but otherwise are poorly correlated to the theoretical results, representing poor fidelity.
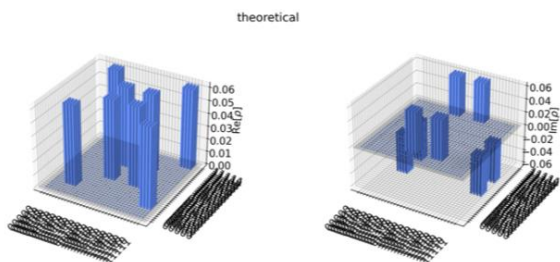

Fig. 5. BBPSSW 3-pass Distillation Theoretical City-State diagram representing the density matrix.
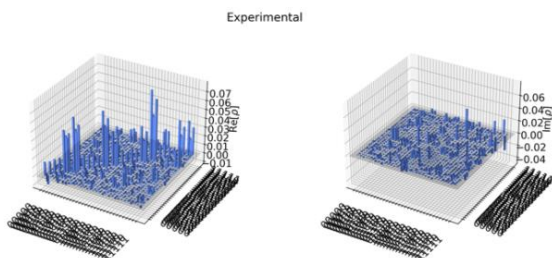

Fig. 6. BBPSSW 3-pass Distillation Experimental City-State diagram representing the density matrix derived via tomography.

Table 2. Distillation Results (Fidelity)

	BBPSSW	DEJMPS	Bell 2q
1 pass	0.15	0.16	0.89
2 pass	0.10	0.12	
3 pass	0.10	0.23	
4 pass	0.08		
5 pass	0.16		

During our testing, fidelity was found to be very low as shown in Table 2, for all the distillation circuits we tested. With

fidelity below 0.5 (a benchmark suggested by the IETF documents), we still observed success at achieving entanglement and passing the data bit. Simple circuits, such as two-qubit Bell entanglement, in comparison, produce high fidelity but are not expected to be adequately resilient to withstand long-distance quantum teleportation.

E. Discussion and Conclusions

We did find, for the BBPSSW implementation, that multi-pass distillation works as anticipated, with successive passes improving the success of transmitting a bit of data up to the 4th pass in this set of tests. Though it is currently not possible to implement with feedback loops, as the criterion for continued distillation passes, we found the process can be studied through post-circuit data analysis.

The multi-pass distillation process demonstrates a good use case, for the reset and reuse functionality. The temporary nature of the auxiliary qubits data provides an excellent opportunity to reset and reuse these qubits. Reset-Reuse allows more efficient use of the qubits and will allow us to implement technology, like repeaters, with fewer qubits required.

In our test results, we notice that fidelity alone may guide us to discard some approaches which show success, based on other metrics, such as entanglement success and success of transmitting a bit of data. The fidelity was experimentally found to be excessively low, for the complex process of multi-pass distillation.

Acknowledgment

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